Numpy tutorial

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... some ideas shamelessly stolen from last year's tutorial by Stefan van der Walt...

CHAPTER

ONE

ADVANCED NUMPY

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c.f. introductory tutorial http://scipy-lectures.github.com/

1.1 The Plan

1. Numpy under the hood
2. Broadcasting
3. Fancy indexing
4. Structured arrays

CHAPTER

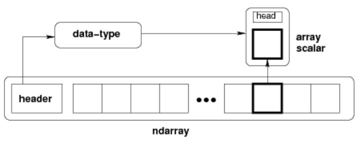
TWO

NUMPY UNDER THE HOOD

2.1 It's...

ndarray =

- block of memory
- how to interpret an element
- how to locate an element




```
typedef struct PyArrayObject {
        PyObject_HEAD

        /* Block of memory */
        char *data;

        /* Data type descriptor */
        PyArray_Descr *descr;

        /* Indexing scheme */
        int nd;
        npy_intp *dimensions;
        npy_intp *strides;

        /* + other stuff */
} PyArrayObject;
```

2.2 Block of memory

```
>>> import numpy as np
>>> x = np.array([1, 2, 3, 4])
>>> x.data
<read-write buffer for 0xa37bfd8, size 16, offset 0 at 0xa4eabe0>
>>> str(x.data)
'\x01\x00\x00\x00\x02\x00\x00\x00\x03\x00\x00\x00\x04\x00\x00\x00'
```

- Memory address

```
>>> x.__array_interface__['data'][0]
140507238089520
```

2.3 Sharing memory

Two `ndarrays` may be **views** to the same memory:

```
>>> x = np.array([1,2,3,4])
>>> y = x[:]
>>> x is y
False
>>> x[0] = 9
>>> y
array([9, 2, 3, 4])
```

The `.base` attribute:

```
>>> y.base
array([9, 2, 3, 4])
>>> y.base is x
True
```

Memory does not need to be owned by an `ndarray`:

```
>>> x = '1234'
>>> y = np.frombuffer(x, dtype=np.byte)
>>> y.data
<read-only buffer for 0xa588ba8, size 4, offset 0 at 0xa55cd60>
>>> y.base is x
True
```

2.4 Flags

```
>>> x = '1'
>>> y = np.frombuffer(x, dtype=np.int8)
>>> y.flags
  C_CONTIGUOUS : True
  F_CONTIGUOUS : True
  OWNDATA : False
  WRITEABLE : False
  ALIGNED : True
  UPDATEIFCOPY : False
```

- The `owndata` and `writeable` flags indicate status of the memory block.
- Some flags can be changed.

```
>>> y.flags.writeable = True
```

- A mathematical detour.

2.5 Data types

```
>>> x.dtype
```

`dtype` describes how to interpret bytes of an item.

Attribute	
itemsize	**size** of the data block
type	*int8*, *int16*, *float64*, etc. (fixed size) *str*, *unicode*, *void* (varying sizes)
byteorder	**byte order**: big-endian > / little-endian < / not applicable |
...	...

```
>>> np.dtype(int).type
<type 'numpy.int32'>
>>> np.dtype(int).itemsize
4
>>> np.dtype(int).byteorder                # Native (little-endian)
'='
```

- Byte order:

```
>>> np.frombuffer('\x01\x02', dtype='>i2')  # Big-endian
array([258], dtype=int16)
>>> 1 * 2**8 + 2 * 2**0
258

>>> np.frombuffer('\x01\x02', dtype='<i2')  # Little-endian
array([513], dtype=int16)
>>> 1 * 2**0 + 2 * 2**8
513
```

2.6 Aside: casting

- Automatic: on assignment, array construction, arithmetic, etc.
- Manually: `.astype(dtype)`
- Makes a **copy**

```
>>> x = np.array([1, 2, 3, 4], dtype=np.float)
>>> x
array([ 1.,  2.,  3.,  4.])
>>> y = x.astype(np.int8)
>>> y
array([1, 2, 3, 4], dtype=int8)
>>> y + 256.0
array([ 257.,  258.,  259.,  260.])
>>> y + np.array([256], dtype=np.int32)
array([258, 259, 260, 261])
```

- Strings work (beware of length):

```
>>> z = y.astype('S8')
>>> z
array(['1', '2', '3', '4'],
      dtype='|S8')
```

```
>>> z.astype(int)
array([1, 2, 3, 4])

>>> x = np.array([100]).astype('S2').astype(int)
>>> x
array([10])
```

- Casting on setitem: dtype of the array **is not** changed on item assignment

```
>>> y[:] = y + 1.5
>>> y
array([2, 3, 4, 5], dtype=int8)
>>> y += 256
array([2, 3, 4, 5], dtype=int8)
```

2.7 Data re-interpretation

Array item (4 bytes)

0x01	‖	0x02	‖	0x03	‖	0x04

- 1 of int32, OR,
- 1 of float32, OR,
- string of length 4, OR,
- ...
- Swap the `dtype` object to a different one:

```
>>> x = np.array(['\x01\x02\x03\x04'], dtype='S4')
>>> y = x.view(np.int32)
>>> y
array([67305985])
>>> hex(67305985)
'0x4030201'
>>> y.dtype
```

2.8 Data re-interpretation

- Viewing with different item size:

```
>>> x = np.array([1, 2, 3, 4], dtype=np.uint8)
>>> y = x.view("<i2")
>>> y
array([ 513, 1027], dtype=int16)
>>> 0x0201, 0x0403
(513, 1027)
>>> str(x.data) == str(y.data)
True
```

0x01	0x02	‖	0x03	0x04

- `.view()` makes *views*, does not copy (or alter) the memory block
- Only changes the dtype (and adjusts array shape)

```
>>> x[1] = 5
>>> y
array([ 1281, 1027], dtype=int16)
>>> y.base is x
True
```

2.9 Not so fast!

- Multidimensional array

```
>>> x = np.array([[1, 2], [3, 4]], dtype=np.uint8)
>>> y = x.T.copy().T
>>> x
array([[1, 2],
       [3, 4]], dtype=uint8)
>>> y
array([[1, 2],
       [3, 4]], dtype=uint8)
>>> x.view(np.int16)
array([[ 513],
       [1027]], dtype=int16)
>>> y.view(np.int16)
array([[ 769, 1026]], dtype=int16)
```

???

2.10 Not so fast!

```
>>> str(x.data)
'\x01\x02\x03\x04'
>>> str(y.data)
'\x01\x03\x02\x04'
```

- But:

```
>>> x
array([[1, 2],
       [3, 4]], dtype=uint8)
>>> y
array([[1, 2],
       [3, 4]], dtype=uint8)
```

- What does `x[0,1]` mean?

2.11 Indexing?

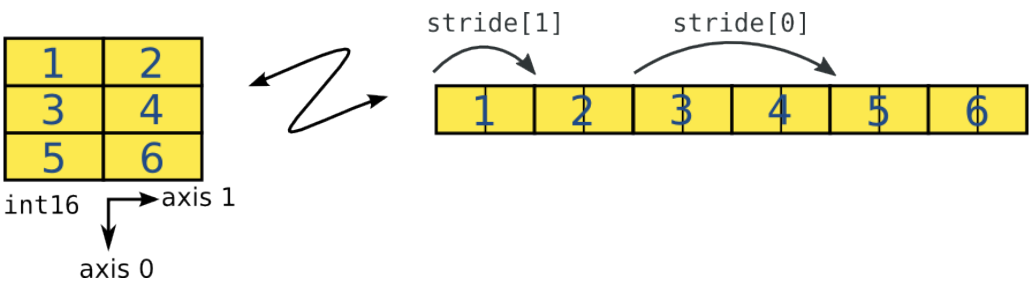
The question

```
>>> x = np.array([[1, 2],
...               [3, 4],
...               [5, 6]], dtype=np.int8)
>>> str(x.data)
'\x01\x02\x03\x04\x05\x06'
```

At which byte in `x.data` does the item `x[2,1]` begin?

2.12 Indexing: strides

The answer (in Numpy)



- **strides**: the number of bytes to jump to find the next element
- 1 stride per dimension

2.13 Indexing: strides (2)

- Looking for `x[2,1]`

```
>>> x = np.array([[1, 2],
...               [3, 4],
...               [5, 6]], dtype=np.int16)
>>> str(x.data)
'\x01\x00\x02\x00\x03\x00\x04\x00\x05\x00\x06'
>>> x.strides
(4, 2)
>>> byte_offset = ...    # x[2,1]
>>> x.data[byte_offset]
'\x06'
>>> x[2,1]
6
```

- simple, **flexible**

2.14 C and Fortran order?

C-order (large strides first, no gaps)

```
>>> x = np.array([[1, 2],
...               [3, 4],
...               [5, 6]], dtype=np.int16, order='C')
>>> x.strides
(4, 2)
>>> str(x.data)
'\x01\x00\x02\x00\x03\x00\x04\x00\x05\x00\x06\x00\x07\x00\x08\x00\x09\x00'
>>> x.flags
```

- Need to jump 4 bytes to find the next row

- Need to jump 2 bytes to find the next column

Fortran-order (small strides first, no gaps)

```
>>> y = np.array(x, order='F')
>>> y.strides
(2, 6)
>>> str(y.data)
'\x01\x00\x03\x00\x05\x00\x02\x00\x04\x00\x06\x00'
>>> y.flags
```

- Need to jump 2 bytes to find the next row
- Need to jump 6 bytes to find the next column

```
>>> x == y
```

2.15 Indexing: Slicing

- **All slicing operations**: just adjust `shape`, `strides` (and `data`)!
- **Never** need to make copies

```
>>> x = np.array([1, 2, 3, 4, 5, 6], dtype=np.int32)
>>> y = x[::-1]
>>> y
array([6, 5, 4, 3, 2, 1])
>>> y.strides
(-4,)

>>> y = x[2:]
>>> y.__array_interface__['data'][0] - x.__array_interface__['data'][0]
8

>>> x = np.zeros((10, 10, 10), dtype=np.float)
>>> x.strides
(800, 80, 8)
>>> x[::2,::3,::4].strides
(1600, 240, 32)
```

2.16 Manual stride manipulation

```
>>> from numpy.lib.stride_tricks import as_strided
>>> as_strided?

>>> x = np.array([1, 2, 3, 4], dtype=np.int16)
>>> x[::2]
>>> x[::2].strides
(4,)
>>> as_strided(x, strides=(4,), shape=(2,))
array([1, 3], dtype=int16)
```

2.17 More strides: diagonals

```
>>> x = np.array([[1, 2, 3],
...               [4, 5, 6],
...               [7, 8, 9]], dtype=np.int32)
```

Q: Pick the diagonal entries

```
>>> x_diag = as_strided(x, shape=(3,), strides=((3+1)*x.itemsize,))
>>> x_diag
array([1, 5, 9])
```

2.18 More strides: A small mistake...

Bad:

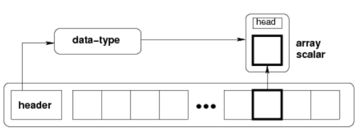
```
>>> x_diag = as_strided(x, shape=(3e6,), strides=((3+1)*x.itemsize,))
>>> x_diag += 9
Segmentation fault (core dumped)
```

Even worse:

```
>>> x_diag = as_strided(x, shape=(4,), strides=((3+1)*x.itemsize,))
>>> x_diag += 9
>>> # <-- No segmentation fault!
```

> **Warning:** `as_strided` does **not** do any sanity checks...
> Good only for:
> - Demonstrating strides.
> - For writing functions that do a specific thing (and make the checks!)

2.19 Summary of internals



- *memory block*: may be shared, `.base`, `.flags`
- *data type descriptor*: what is in each data cell, casting, `.view()`
- *indexing*: strides, C/F-order, slicing, `as_strided`, some stride tricks

CHAPTER

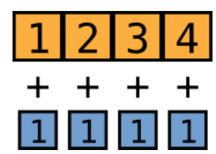
THREE

EVERYDAY FEATURES: BROADCASTING

3.1 Scalars

- Scalars add elementwise:

```
>>> np.array([1, 2, 3, 4]) + 1
array([2, 3, 4, 5])
```

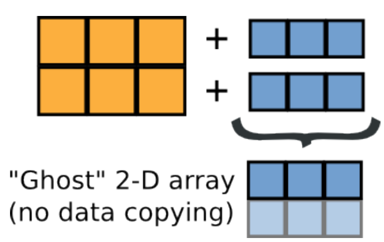


- Same for other binary operations (and many other functions).

3.2 Arrays?

- Broadcasting: **arrays behave like scalars along an axis**

```
>>> a = np.array([[10, 20, 30], [40, 50, 60]])
>>> b = np.array([1, 2, 3])
```




```
>>> c = a + b
>>> c
array([[11, 22, 33],
       [41, 52, 63]])
```

-

$$c_{ij} = a_{ij} + b_j$$

```
>>> a[1,2] + b[2] == (a + b)[1,2]
True
```

3.3 Shape matching

- Same number of dimensions:

```
>>> a = np.array([[10, 20, 30], [40, 50, 60]])
>>> b = np.array([1, 2, 3])[np.newaxis,:]
>>> c = a + b
>>> a.shape
(2, 3)
>>> b.shape
(1, 3)
>>> c.shape
(2, 3)
```

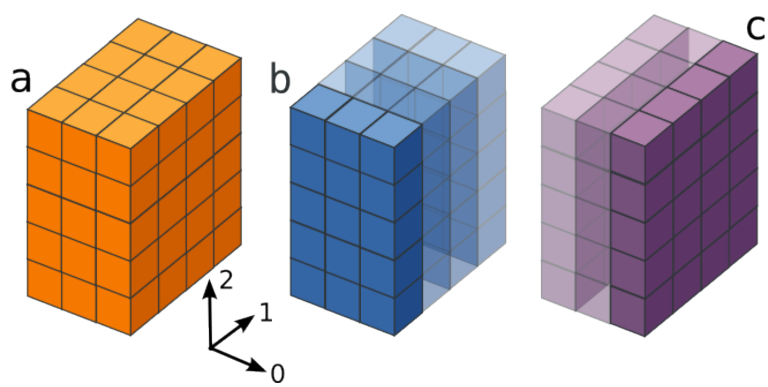
Shape arithmetic:

```
  (2, 3)          (2, 3)
  (1, 3)             (3,)      <-- behaves as scalar for axis=0
--------        --------
  (2, 3)          (2, 3)
```

3.4 Higher dimensions: more of the same

```
>>> a = np.random.rand(3, 4, 5)
>>> b = np.random.rand(3, 1, 5)
>>> c = np.random.rand(4, 5)
```

```
(3, 4, 5)          (3, 4, 5)
(3, 1, 5)             (4, 5)
---------          ---------
(3, 4, 5)          (3, 4, 5)
```



-

$$(a - b)_{ijk} = a_{ijk} - b_{ik}$$

```
>>> (a[1,3,2] - b[1,0,2]) == (a - b)[1,3,2]
True
```

- $$(a - c)_{ijk} = a_{ijk} - c_{jk}$$

```
>>> (a[1,3,2] - c[3,2]) == (a - c)[1,3,2]
True
```

3.5 Common uses (1/3)

- Evaluating something on a grid:

```
>>> x, y = np.arange(5), np.arange(5)
>>> distance = np.sqrt(x**2 + y[:, np.newaxis]**2)
>>> distance
array([[ 0.        ,  1.        ,  2.        ,  3.        ,  4.        ],
       [ 1.        ,  1.41421356,  2.23606798,  3.16227766,  4.12310563],
       [ 2.        ,  2.23606798,  2.82842712,  3.60555128,  4.47213595],
       [ 3.        ,  3.16227766,  3.60555128,  4.24264069,  5.        ],
       [ 4.        ,  4.12310563,  4.47213595,  5.        ,  5.65685425]])
```

3.6 Common uses (2/3)

- `np.ogrid`

```
>>> x, y = np.ogrid[0:5, 0:5]
>>> x.shape, y.shape
((5, 1), (1, 5))
>>> distance = np.sqrt(x**2 + y**2)
```

- `np.ix_`

3.7 Common uses (3/3)

- Tensor operations

Example: many matrix products for small matrices

```
>>> R = np.random.rand(3, 3, 2000) # 2000 of 3x3 matrices
>>> Z = np.random.rand(3, 3, 2000)
```

Compute `R_k.dot(Z_k)` for each 3x3 matrices `R_k` and `Z_k` in `R`, `Z`

$(RZ)_{ijk} = \sum_p R_{ipk} Z_{pjk}$

```
    i    p    j    k
R   :    :    na   :
Z   na   :    :    :
---------------------
RZ  :   sum   :    :
```

```
>>> RZ = (R[:,:,newaxis,:] * Z[newaxis,:,:,:]).sum(axis=1)
```

- ... or `einsum` (Numpy >= 1.6; faster)

```
>>> ...
```

3.8 Explicit broadcasting

- Explicitly broadcast arrays are sometimes useful:

```
>>> x = np.array([10, 20, 30, 40]).reshape(1, 4)
>>> y = np.array([1, 2, 3]).reshape(3, 1)

>>> x2, y2 = np.broadcast_arrays(x, y)
>>> x2
array([[10, 20, 30, 40],
       [10, 20, 30, 40],
       [10, 20, 30, 40]])
>>> y2
array([[1, 1, 1, 1],
       [2, 2, 2, 2],
       [3, 3, 3, 3]])
```

- They're views?

```
>>> x[0,0] = -1
>>> x2
array([[-1, 20, 30, 40],
       [-1, 20, 30, 40],
       [-1, 20, 30, 40]])
```

- Strides?

```
>>> ...
```

3.9 Ghost arrays

- Internally, broadcasting uses **0-strides**

```
>>> x = np.array([10, 20, 30, 40]).reshape(1, 4)
>>> y = np.array([1, 2, 3]).reshape(3, 1)

>>> from numpy.lib.stride_tricks import as_strided

>>> x2 = as_strided(x, strides=(...), shape=(3, 4)) # No copying!
>>> x2
array([[10, 20, 30, 40],
       [10, 20, 30, 40],
       [10, 20, 30, 40]])

>>> y2 = as_strided(y, strides=(...), shape=(3, 4))
>>> y2
array([[1, 1, 1, 1],
       [2, 2, 2, 2],
       [3, 3, 3, 3]])
```

- Indexing vs. 0-strides

```
>>> byte_offset = ...
```

CHAPTER

FOUR

EVERYDAY FEATURES: FANCY INDEXING

4.1 Boolean masks

```
>>> a = np.array([1, 2, 3, 4])
>>> a > 2
array([False, False,  True,  True], dtype=bool)
>>> a[a > 2]
array([3, 4])
```

- Always a copy (cannot do with strides)

```
>>> a[a > 2][0] = -1     # OBS!
>>> a
array([1, 2, 3, 4])
```

- Assignment works:

```
>>> a[a > 2] = -1
>>> a
array([ 1,  2, -1, -1])
```

4.2 Boolean masks, ndim > 1

- 1-D masks

```
>>> a = np.arange(4*5).reshape(4, 5)
>>> a
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
```

- Extract rows

```
>>> a[np.array([True,False,False,True])]
array([[ 0,  1,  2,  3,  4],
       [15, 16, 97, 98, 99]])
```

- Extract columns

```
>>> a[:, np.array([True,True,False,False,True])]
array([[ 0,  1,  4],
       [ 5,  6,  9],
       [10, 11, 14],
       [15, 16, 99]])
```

4.3 Boolean masks, ndim > 1

- `mask.ndim == arr.ndim`: **result is 1-D**

```
>>> a = np.arange(4*5).reshape(4, 5)
>>> a[a > 16]
array([17, 18, 19])
```

- Makes this possible:

```
>>> mask = (a > 16)
>>> b = a[mask] + 80
>>> a[mask] = b
>>> a
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 97, 98, 99]])
```

- Or this:

```
>>> b = np.zeros_like(a)
>>> b[mask] = a[mask]
>>> b
array([[ 0,  0,  0,  0,  0],
       [ 0,  0,  0,  0,  0],
       [ 0,  0,  0,  0,  0],
       [ 0,  0, 97, 98, 99]])
```

4.4 Integer indexing

- **In a nutshell**:

  ```
  a = 2-dim array
  p = integer array of shape (M, N, K)
  q = integer array of shape (M, N, K)

  b = a[p, q]
  ```

  produces `b`:

  ```
  b.shape == (M, N, K)

  b[i,j,k] == a[p[i,j,k], q[i,j,k]]
  ```

- `p` and `q` are broadcast first (against each other)
- Similarly for all # of dimensions.

4.5 Integer indexing, simple

```
>>> a = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
```

- Pick diagonal:

```
>>> i = np.arange(3)
>>> a[i,i]
array([1, 5, 9])
```

- Pick 2x2 block

```
>>> ... # ix_
```

4.6 Integer indexing + slices

- Mix with slices: pick rows:

```
>>> a[[0, 2]]
array([[1, 2, 3],
       [7, 8, 9]])
```

- Pick columns:
- Higher dimensions...

```
>>> a = np.arange(3*4*5).reshape(3,4,5)
>>> i = np.array([0, 1])
>>> j = np.array([1, 2])
>>> a[:,i,j][:,0]
array([ 1, 21, 41])
>>> a[:,i[0],j[0]]
array([ 1, 21, 41])
```

OK...

```
>>> a[i,:,j][:,0]
array([ 1,  22])
>>> a[i[0],:,j[0]]
array([ 1,  6, 11, 16])
```

What?

4.7 Integer indexing + slices

- That is:

```
a = 4-dim array of shape (p, q, r, s)
II = integer array of shape (M, N, K)
JJ = integer array of shape (M, N, K)

b = a[:, II, JJ, :]

c = a[:, II, :, JJ]
```

produces `b`, `c`:

`b.shape == (p, M, N, K, q)`

`b[i,j,k,l,m] == a[i, II[j,k,l], JJ[j,k,l], m]`

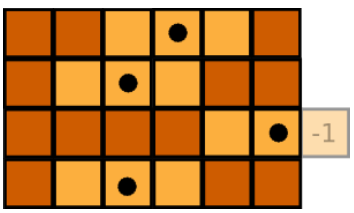
`c.shape == (M, N, K, p, q)`

`c[i,j,k,l,m] == a[l, II[i,j,k], m, JJ[i,j,k]]`

- Fancy indices are **next to each other**: *fancy axes* go to the **same position**
- Otherwise, *fancy axes* go **first**

4.8 Windows to data

Pick the largest value from each row on a 2-D array, and its 2 neighbors. (Produce `N x 3` array of results, mark 'missing' data with -1.)



Some "data":

```
>>> a = np.random.zipf(1.3, size=(10, 5))
>>> a
array([[    1,  1339,   113,     1,     3],
       [    3,    27,    63,     6,     1],
       [    3,    14,     1,     1,     2],
       [ 1046,     1,     1,    66,     1],
       [   14,     2,     9,     1, 39633],
       [    4,   136,   258,    27,     1],
       [  661,    11,   313,     4,     1],
       [   55,    55,     1,    13,    72],
       [    1,     5,  1027,    12,   134],
       [  214,    11,     3,   274,     1]])
```

Locate maximum:

```
>>> j_max = np.argmax(a, axis=1)
```

Generate 2-D fancy-index arrays:

```
>>> i = np.arange(a.shape[0])[:,np.newaxis]
>>> j = j_max[:,np.newaxis] + np.array([-1, 0, 1])

>>> i, j = np.broadcast_arrays(i, j)
>>> i.shape
(10, 3)
>>> j.shape
(10, 3)
```

Result array:

```
>>> b = np.zeros((a.shape[0], 3), dtype=a.dtype)
>>> b[...] = -1  # marker for no neighbor
```

Mask out invalid indices (biggest number can be at the edge):

```
>>> mask = (j >= 0) & (j < a.shape[1])
```

Fancy stuff:

```
>>> b[mask] = a[i[mask], j[mask]]
```

Result:

```
>>> a
array([[    1,  1339,   113,     1,     3],
       [    3,    27,    63,     6,     1],
       [    3,    14,     1,     1,     2],
       [ 1046,     1,     1,    66,     1],
       [   14,     2,     9,     1, 39633],
       [    4,   136,   258,    27,     1],
       [  661,    11,   313,     4,     1],
       [   55,    55,     1,    13,    72],
       [    1,     5,  1027,    12,   134],
       [  214,    11,     3,   274,     1]])
>>> b
array([[    1,  1339,   113],
       [   27,    63,     6],
       [    3,    14,     1],
       [   -1,  1046,     1],
       [    1, 39633,    -1],
       [  136,   258,    27],
       [   -1,   661,    11],
       [   13,    72,    -1],
       [    5,  1027,    12],
       [    3,   274,     1]])
```

CHAPTER

FIVE

EVERYDAY FEATURES: STRUCTURED DATA TYPES

5.1 Composite data

```
% sensor  position  measurement
ALFA      1.1       1.4
BETA      1.3       14
TAU       1.5       -3
BETA      1.4       18
...
```

sensor_code	4-character string
position	float
value	float

5.2 Structured type

sensor_code	4-character string
position	float
value	float

```
>>> samples = np.zeros((3,), dtype=[('sensor_code', 'S4'),
...                                  ('position', float), ('value', float)])
>>> samples.dtype.itemsize
12
>>> samples.ndim
1
>>> samples.shape
(3,)
>>> samples.dtype.names
('sensor_code', 'position', 'value')

>>> samples[:] = [('ALFA', 1, 0.35), ('BETA', 1, 0.11), ('ALFA', 1.5, 0.35)]
>>> samples
array([('ALFA', 1.0, 0.35), ('BETA', 1.0, 0.11), ('ALFA', 1.5, 0.35)],
      dtype=[('sensor_code', '|S4'), ('position', '<f8'), ('value', '<f8')])
```

Note: Other syntaxes for structured arrays: http://docs.scipy.org/doc/numpy/user/basics.rec.html

5.3 Loading from a text file

```
% sensor  position  measurement
ALFA   1.1  1.4
BETA   1.3  14
TAU    1.5  -3
BETA   1.4  18
...
```

```
>>> sensor_dtype = np.dtype([('sensor_code', 'S4'),
...                          ('position', float), ('value', float)])
>>> samples = np.loadtxt('sensordata.txt', dtype=sensor_dtype, comments='%')
>>> samples
array([('ALFA', 1.1, 1.4), ('BETA', 1.3, 14.0), ('TAU', 1.5, -3.0),
       ('BETA', 1.4, 18.0), ('TAU', 1.6, -2.0), ('BETA', 1.5, 14.0),
       ('TAU', 1.7, -3.0), ('ALFA', 2.2, 1.8), ('TAU', 1.9, -2.0)],
      dtype=[('sensor_code', '|S4'), ('position', '<f8'), ('value', '<f8')])
```

5.4 Accessing fields

- Index with field names:

```
>>> samples['sensor_code']
array(['ALFA', 'BETA', 'TAU', 'BETA', 'TAU', 'BETA', 'TAU', 'ALFA', 'TAU'],
      dtype='|S4')
>>> samples['value']
array([  1.4,  14. ,  -3. ,  18. ,  -2. ,  14. ,  -3. ,   1.8,  -2. ])
```

- It's a view, not copy:

```
>>> samples['value'].base is samples
True
```

- Multiple fields at once: (NB: `['position', 'value']` is a list)

```
>>> samples[['position', 'value']]
array([(1.1, 1.4), (1.3, 14.0), (1.5, -3.0), (1.4, 18.0), (1.6, -2.0),
       (1.5, 14.0), (1.7, -3.0), (2.2, 1.8), (1.9, -2.0)],
      dtype=[('position', '<f8'), ('value', '<f8')])
```

5.5 Indexing &c.

- Everything else works as usual!

```
>>> samples[0]
('ALFA', 1.0, 0.35)

>>> samples[0]['sensor_code'] = 'TAU'
>>> samples[0]
('TAU', 1.0, 0.35)

>>> samples[samples['sensor_code'] == 'ALFA']
array([('ALFA', 1.0, 0.35), ('ALFA', 1.5, 0.35), ('ALFA', 2.1, 0.11)],
      dtype=[('sensor_code', '|S4'), ('position', '<f8'), ('value', '<f8')])
```

5.6 Some utility functions

- np.rec

```
>>> a = np.array([1,2,3])
>>> b = np.array(['a','b','c'])
>>> c = np.rec.fromarrays([a, b], names='position,name')
rec.array([(1, 'a'), (2, 'b'), (3, 'c')],
      dtype=[('position', '<i8'), ('name', '|S1')])
```

- NB: These return an array subclass

```
>>> type(c)
numpy.core.records.recarray
>>> c.position, c['position']
(array([1, 2, 3]), array([1, 2, 3]))
```

- Converting back to an usual array:

```
>>> c = c.view(np.ndarray)
```

5.7 Re-interpreting data as structured arrays

You have RGB image data in an array

```
>>> x = np.zeros((10, 10, 3), dtype=np.int8)
>>> x[:,:,0] = 1
>>> x[:,:,1] = 2
>>> x[:,:,2] = 3
```

where the last three dimensions are the R, B, and G channels.

How to make a (10, 10) structured array with field names 'r', 'g', 'b', 'a' without copying data?

```
>>> y = ...

>>> assert (y['r'] == 1).all()
>>> assert (y['g'] == 2).all()
>>> assert (y['b'] == 3).all()
>>> assert (y['a'] == 4).all()
```

5.8 Re-interpreting data as structured arrays

```
>>> rgb_dtype = np.dtype([('r', 'i1'),
...                       ('g', 'i1'),
...                       ('b', 'i1')])
>>> y = x.view(rgba_dtype)[:,:,0]
```

- Beware:

```
>>> assert y.flags.c_contiguous
```

CHAPTER

SIX

SUMMARY

- Internals

  Indexing, slicing, strides, etc.
- Broadcasting
- Fancy indexing
- Structured arrays

CHAPTER

SEVEN

EXERCISES

7.1 Setup

To kit up, launch IPython:

```
ipython
```

and import Numpy:

```
In [1]: import numpy as np
```

Tune how it prints arrays (easier for the eyes):

```
In [2]: np.set_printoptions(precision=3)
```

7.2 Warming up

7.2.1 Exercise 1: Warming up

1. Create a `5x6` Numpy array containing random numbers in range `[0, 1]`.
   - Compute the mean of all the numbers in it

     (To find the function to do this: `np.lookfor("mean of array")`)
   - Compute the minimum value in each row, and maximum in each column
   - Multiply each element by `10` and convert to an integer with the `.astype()` method.

     What is the difference between `a.astype(int)` and `np.around(a)`?
2. Compare:

   ```
   np.array([1, 2, 3, 4]) / 2
   np.array([1.0, 2, 3, 4]) / 2
   np.array([1, 2, 3, 4]) // 2
   np.array([1.0, 2, 3, 4]) // 2
   ```

   Why does it work like it does? How about with:

   ```
   a = np.array([1, 2, 3, 4], dtype=float)
   b = np.array([1.0, 2.0, 3.0, 4.0], dtype=np.int8)
   ```
3. Which of the following operations create a view, and which a copy:

```
a = np.array([[1, 2, 3], [4, 5, 6]])

a[:,[0,1]]
a[:,0:2]
a[0]
a.T
a[[True, False]]

a.reshape(2*3)             # bonus sector
a.T.reshape(2*3)           # bonus sector
```

(Think first, then check.)

4. Use the function:

```
def change_it(x):
    x[:] = np.array([7, 8, 9])
```

to change array:

```
a = np.array([1, 2, 3, 4, 5, 6])
```

to:

```
array([1, 7, 8, 9, 5, 6])
```

7.2.2 Exercise 2: Strides

1. Consider:

```
a = np.array([[1, 2, 3], [4, 5, 6]], dtype=np.int16)
```

What do the following operations do, and what are the resulting strides:

```
a
a.T
a[::-1]
```

2. Study the `.strides`, `.flags`, and `str(a.data)` attributes of the arrays:

```
a = np.array([[1, 2], [3, 4]], dtype=np.byte)
b = a.T
```

Which of the above are C-contiguous (and what does that mean)?

7.3 Broadcasting

7.3.1 Exercise 3: Operating along an axis

Divide each column of the array

```
>>> a = np.arange(25).reshape(5, 5)
>>> a
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
```

```
       [15, 16, 17, 18, 19],
       [20, 21, 22, 23, 24]])
```

elementwise with the array `b = np.array([1.5, 5, 10, 15, 20])`.

I.e., the result should be:

```
array([[ 0/1.5,  1/1.5, ...],
       [ 5/5,    6/5,   ...],
       [10/10,  11/10,  ...],
       [15/15,  16/15,  ...],
       [20/20,  21/20,  ...]])
```

Tips

- `np.newaxis`

7.3.2 Exercise 4: Integral approximation

Write a function `f(a, b, c)` that returns $a^b - c$. Generate a shape `(24, 12, 6)` array containing the values `f(a_i, b_j, c_k)` at points *a_i*, *b_j* and *c_k* forming a grid in the unit cube `[0, 1] x [0, 1] x [0, 1]`.

Approximate the 3-d integral

$$\int_0^1 \int_0^1 \int_0^1 (a^b - c) da\, db\, dc$$

over this volume with the mean of the values. The exact result is: $\log(2) - \frac{1}{2}$ — how close do you get?

Try also using `np.mgrid` instead of broadcasting. Is there a speed difference? How about `ogrid` with `broadcast_arrays`?

Tips

- You can make `np.ogrid` give a number of points in given ranges with the syntax `np.ogrid[a:b:20j, c:d:10j]`.
- You can use `%timeit` in IPython to check timings

7.4 Fancy indexing

7.4.1 Exercise 5: Picking up

1. Extract the 1st superdiagonal `1, 7, 14` from the array:

   ```
   0   1  2  3
   5   6  7  8
   11 12 13 14
   15 16 17 18
   19 20 21 22
   ```

   Then extract the 1st and the 3rd columns.

2. Generate a 10 x 3 array of random numbers (in range [0,1]). From each row, pick the number closest to 0.75.

> **Tips**
>
> - Make use of `np.abs` and `np.argmax` to find the column `j` closest for each row.
> - Use fancy integer indexing to extract the numbers. Remember that in `a[i,j]` the index array `i` must correspond to `j`.

7.5 Structured data types

7.5.1 Exercise 6: Basic handling

Design a structured data type suitable for the data (in `words.txt`):

```
% rank          lemma (10 letters max)      frequency       dispersion
21              they                        1865844         0.96
42              her                         969591          0.91
49              as                          829018          0.95
7               to                          6332195         0.98
63              take                        670745          0.97
14              you                         3085642         0.92
35              go                          1151045         0.93
56              think                       772787          0.91
28              not                         1638883         0.98
```

Load the data from the text file. Examine the data you got, for example: extract words only, extract the 3rd row, print all words with rank `< 30`.

Sort the data according to frequency. Save the result to a Numpy data file `sorted.npz` with `np.savez` and load back with `np.load`. Do you get back what you put in?

Save the result to a text file `sorted.txt` using `np.savetxt`. Here, you need to provide a `fmt` argument to `savetxt`.

> **Tips**
>
> - See the documentation of the `.sort()` method: `help(np.ndarray.sort)`
> - For structured arrays, `savetxt` needs a `fmt` argument that tells it what to do.
>   `fmt` is a string. For example `"%s %d %g"` tells that the first field is to be formatted as a string, the second as an integer, and the third as a float.

7.5.2 Exercise 7: Reading binary files

The `.wav` audio files are binary files: they contain a fixed-size header followed by raw sound data.

Construct a Numpy structured data type describing the `.wav` file header, and use it to read the header. Print for example the sample rate and number of channels. (A `test.wav` is provided so you can try things out on that.)

Tips

- You can read a binary structure described by `some_dtype` to a Numpy array with:

```
with open('test.wav', 'rb') as f:
    data = np.fromfile(f, dtype=some_dtype, count=1)
```

`.wav` file structure

Byte #	Field	
0	chunk_id	4-byte string ("RIFF")
4	chunk_size	4-byte uint (little-endian)
8	format	4-byte string ("WAVE")
12	fmt_id	4-byte string ("fmt ")
16	fmt_size	4-byte uint (little-endian)
20	audio_fmt	2-byte uint (little-endian)
22	num_channels	2-byte uint (little-endian)
24	sample_rate	4-byte uint (little-endian)
28	byte_rate	4-byte uint (little-endian)
32	block_align	2-byte uint (little-endian)
34	bits_per_sample	2-byte uint (little-endian)
36	data_id	4-byte string ("data")
40	data_size	4-byte uint (little-endian)

- `data_size` bytes of actual sound data follow

7.6 Advanced

7.6.1 Exercise A: Indexing

Reimplement array indexing (for 2-D, without using Numpy)! Write a function `data_at_index(indices, data, strides, dtype)` that returns the data corresponding to a specified array element, as a string of bytes. I.e.:

```
a = np.array([[1, 2, 3], [4, 5, 6]], dtype=np.int16)
b = a.T

data_at_index([0, 1], str(a.data), a.strides, a.dtype) == '\x02\x00' == str(a[0,1].data)
data_at_index([0, 1], str(b.data), b.strides, b.dtype) == '\x04\x00' == str(b[0,1].data)
```

Check first that you understand the meaning of:

- the `strides` and `data` attributes of Numpy arrays
- the `type` and `itemsize` attributes of the data type objects

7.6.2 Exercise B: Sliding window

1. Build a sliding 3-item window for the array:

```
x = np.arange(10, dtype=np.int32)
```

The aim is to get an array:

```
array([[0, 1, 2],
       [1, 2, 3],
       [2, 3, 4],
       [3, 4, 5],
       [4, 5, 6],
       [5, 6, 7],
       [6, 7, 8],
       [7, 8, 9]], dtype=int32)
```

without making copies (so that it is fast). The trick is a stride trick:

```
from numpy.lib.stride_tricks import as_strided
strides = ...
y = as_strided(x, shape=(8, 3), strides=strides)
```

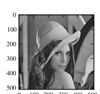
2. Use the same trick to compute the `5 x 5` median filter of an image. For each pixel, compute the median of the `5 x 5` block of pixels surrounding it.

   The median filter provides a degree of denoising similarly to a gaussian blur, but it preserves sharp edges better.

```
>>> import scipy
>>> import matplotlib.pyplot as plt
```

Noisy image

```
>>> img = scipy.lena()  # A standard test image for image processing
>>> img += 0.8 * img.std() * np.random.rand(*img.shape)
>>> plt.imshow(img)
```



Prepare the sliding window

```
>>> assert img.flags.c_contiguous   # Important!

>>> window_size = 5
>>> shape = ...   # Careful, no out-of-bounds access...
>>> strides = ...

>>> img_window = as_strided(...)
```

Denoise!

```
>>> img_median = np.median(img_window.reshape(..., window_size*window_size), axis=...)
>>> plt.imshow(img_median)
>>> plt.gray()
>>> plt.imsave('sharpened.png', img_median)
```

Note:

- Above, the `.reshape()` makes a copy (why?).
- Scipy has an implementation for the median filter in `scipy.ndimage`, with more features.

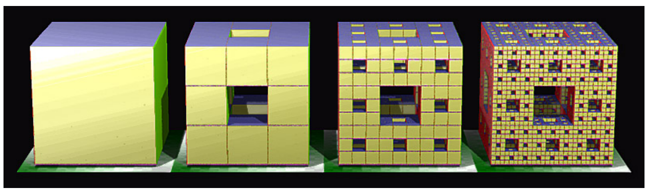
Extra: Visit the factory

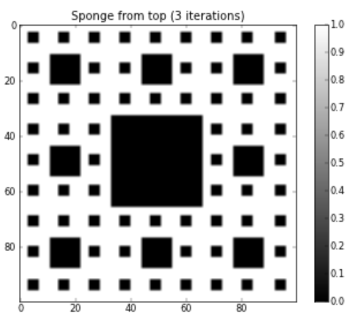
We don't yet have a `rolling_window` function in Numpy that would make the above easier. We, however, do have a contributed implementation that is discussed here:

> https://github.com/numpy/numpy/pull/31

Can you extend the version posted by Warren to make N-dimensional windows, or think of any other features such a function would need to have? (If yes, just ask me how to contribute your stuff.)

7.6.3 Exercise C: Menger sponge



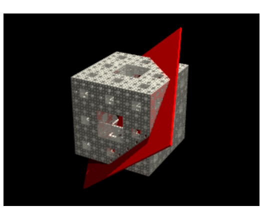


Generate an approximation to the Menger sponge by creating a 3-D Numpy array filled with *1*, and drilling holes to it with slicing.

Tips:

- Use dtype `np.int8` so you don't eat all memory
- Power-of-3 size cube works best, e.g., `81 x 81 x 81`
- You need a function to recurse to drill many levels
- `s = np.s_[i:j]` creates a "free" slice object: `a[s] == a[i:j]`.

Take a 2-D slice of the sponge diagonally through the center of the cube, with normal vector `(1, 1, 1)`. What sort of a patterns you get in the intersection?



Tips (for one approach):

- Fancy indexing with three 2-D integer arrays can give the slice
- Boolean mask helps to exclude out-of-bounds indices
- Vectors `u = np.array([0, 1, -1])/1.414` and `v = np.array([1, -0.5, -0.5])/1.225` (orthogonal to `[1, 1, 1]`) can be used as -the basis for the 2-D index arrays.
- `x.astype(int)` converts float arrays to integer arrays

Spoilers:

http://www.nytimes.com/2011/06/28/science/28math-menger.html